The Hong Kong University of Science and Technology

Department of Mathematics

MPhil THESIS EXAMINATION

Studies on Mild Solutions of Stochastic Heat Equations on Riemannian Manifolds Associated to Higher-Order Elliptic Operators

By

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ABSTRACT

In this thesis, we study an initial value problem of stochastic heat equation

$$\begin{cases} \partial_t u(t,\xi) = Hu(t,\xi) + b(\xi, u(t,\xi)) + \sigma(\xi, u(t,\xi))\dot{W}(t,\xi), & t > 0, \xi \in \mathbb{M} \\ u(0,\xi) = u_0(\xi), & \xi \in \mathbb{M}. \end{cases}$$

where H is a certain $2m$ ($m \in \mathbb{N}$) order elliptic operator b and σ are functions of ξ and $u = \{u(\xi)\}_{\xi \in \mathbb{M}}$, $\dot{W}$ is *formally* a space-time white noise on $\mathbb{M}$, and $\mathbb{M}$ is a compact, connected, and smooth Riemannian manifold of dimension N without boundary. We study a mild solution of stochastic heat equation on a higher dimensional Riemannian manifold. Specifically, we extend Funaki's main theorem [4] to a higher dimensional Riemannian manifold based on Davies's heat kernel estimate. As it turns out, Davies's method is functional analytic enough to fit the above situation regardless of the state space. We also show that the resulting mild solution obtained by this approach has "nice" properties.

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Time : **2:30 p.m.**

Venue : **Online via ZOOM**
ID: 783 8058 7880 (Passcode: bveS2a)
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).